ABSOLUTE PERFORMANCE OF DSD MODELS IN FITTING 2DVD MEASUREMENTS FROM GPM GROUND VALIDATION CAMPAIGNS

PMM 2016 Science Team Meeting
Houston, TX, 24 – 26 October 2016

E. Adirosi[1], E. Volpi[2], F. Lombardo[2], and L. Baldini[1]

[1]Institute of Atmospheric Sciences and Climate, CNR, elisa.adirosi@artov.isac.cnr.it, [2]Dipartimento di Ingegneria, Università degli Studi Roma Tre, elena.volpi@uniroma3.it

Motivation

- Modelling raindrop size distribution (DSD) is fundamental to develop reliable precipitation products.
- Gamma distribution is the most widely used but other 2-parameter distributions have been proposed.
- **At what extent assumptions of Gamma and other models are supported by 2DVD measurements ?**

Description of work

- **Gamma, lognormal, and Weibull** distributions (2 parameter) are considered
- Their **absolute statistical performance** in representing DSDs in nature is evaluated.
- **Conditions under which a model is more appropriate** to represent natural DSDs are investigated

Methods

1. DSD definitions

a) Standard definition

Product of concentration of raindrops in a volume of air n_c by the **probability distribution of drop size in the unit volume of air $f_v(D)$** $(V = A\,\Delta t\,v(D))$ where Δt is the sampling time interval, A is the measuring area and $v(D)$ is the terminal fall velocity of drops) : $N(D) = n_c\, f_v(D)$

b) Disdrometer measured

Product of the **probability density function (pdf) of drop diameters at ground $f(D)$** by the number M of drops collected at ground

$f(D)$ and $f_v(D)$ are transformations of one another, if drop terminal velocity – size relation $v(D)$ is known.

Depending on the $v(D)$ functional form, $f(D)$ and $f_v(D)$ could be better described by different models.

2. Statistical inference of $f(D)$ and $f_v(D)$

Gamma, lognormal, and Weibull distributions are fitted to the 2DVD measured drop size spectra by the **Maximum Likelihood Method** (ML):

$$a)\quad \mathcal{L}(\beta,\gamma) = \prod_{i=1}^{M}[p(D_i;\beta,\gamma)]$$

$$b)\quad \mathcal{L}(\beta,\gamma) = \prod_{i=1}^{M}[p(D_i;\beta,\gamma)]^{N_i}$$

where β and γ are the scale and shape parameters and N_i is given by the inverse of the volume of air (V).

3. Model testing

The **Kolmogorov-Smirnov** (KS) **test** is used: a model assumption is accepted if

$$D_M < \Delta_M(\alpha)$$

where $\Delta_M(\alpha)$ is a critical reference value computed through Monte Carlo simulations and

$$D_M = max_i\left|F(D_i) - \hat{F}(D_i)\right|$$

For $f_v(D)$ fitting:

$$\hat{F}_V(D_i) = \frac{1}{\sum_{z=1}^{M} 1/v(D_z)} \sum_{j=1}^{i} \frac{1}{v(D_j)}$$

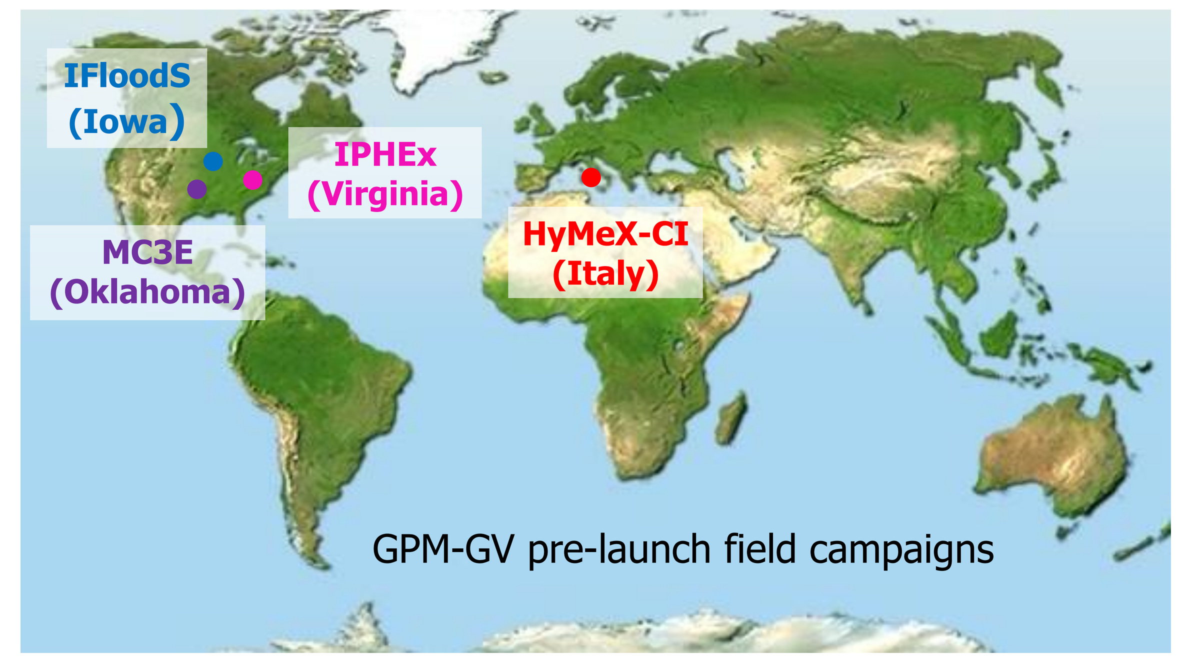
For $f(D)$ fitting:

$\hat{F}_V(D_i)$ is computed with the Weibull plotting position formula

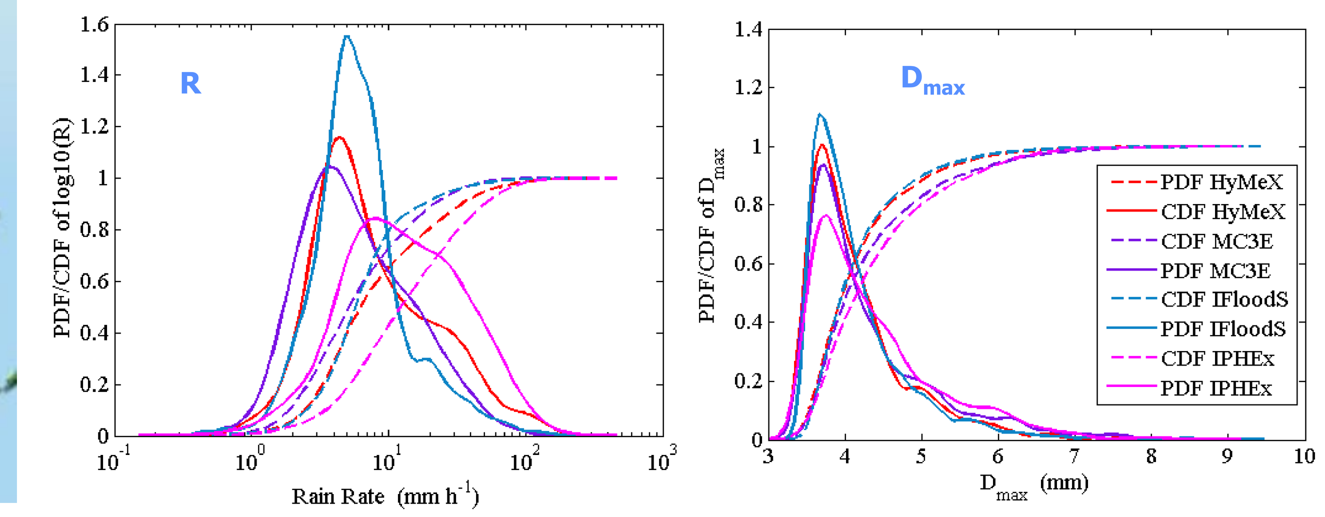
Experimental data

The 2D videodisdrometer (2DVD) is an optical disdrometer that measures the equivolumetric diameter and fall velocity of each single hydrometeor that falls through its virtual measuring area.

	HyMeX	MC3E	IFloodS	IPHEx
# of 1-min samples	2849	6647	22125	10347
max(R) [mm h⁻¹]	158.2	97.6	195.2	194.0
mean(R) [mm h⁻¹]	4.0	2.6	2.6	4.1
max(D_{max}) [mm]	7.79	8.61	9.18	8.65
mean(D_{max}) [mm]	2.54	2.48	2.26	2.27
median(M)	339	299	378	358



Empirical CDF and PDF of R and D_{max} for the four datasets



Results

Rejection rate from KS test (all datasets)

Fitting of $f(D)$	HyMeX	MC3E	IFloodS	IPHEx
gamma	69.0%	66.2%	71.8%	67.0%
lognormal	69.8%	69.6%	80.0%	73.5%
Weibull	81.6%	78.4%	79.5%	78.0%

Fitting of $f_v(D)$	HyMeX	MC3E	IFloodS	IPHEx
gamma	77.3%	73.9%	83.7%	76.7%
lognormal	81.3%	78.9%	88.9%	82.3%
Weibull	85.5%	82.2%	85.9%	82.3%

Success rate (all datasets)

Percentage of samples that have passed the KS test and best fitted by a model (distribution with maximum log-likelihood value is the one that performs best). Completed ML is shown because of negligible differences with truncated ML.

Fitting of $f(D)$	HyMeX	MC3E	IFloodS	IPHEx
gamma	22.1%	22.0%	21.0%	22.8%
lognormal	14.3%	15.1%	8.1%	10.7%
Weibull	9.9%	11.6%	12.2%	13.8%
none	53.6%	51.3%	58.8%	52.6%

Fitting of $f_v(D)$	HyMeX	MC3E	IFloodS	IPHEx
gamma	15.8%	16.7%	11.5%	16.0%
lognormal	10.7%	12.7%	5.3%	8.0%
Weibull	7.3%	8.6%	8.6%	11.1%
none	66.2%	62.0%	74.6%	64.9%

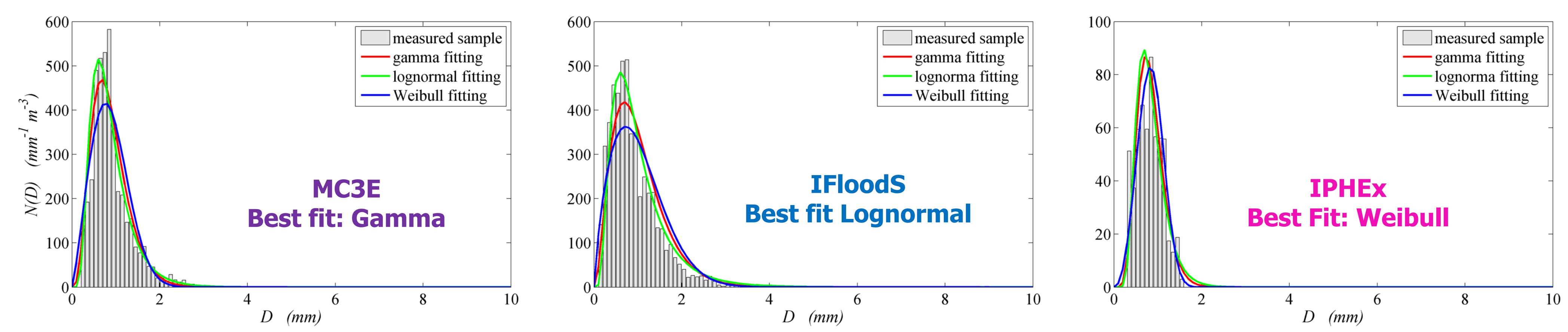
- ✓ For $f_v(D)$ fitting, the gamma distribution is the best …
- ✓ but there is a number of samples that are best fitted by a heavy-tailed distribution (i.e. lognormal distribution).

Percentage of samples that cannot be represented represented by any of the three models (all datasets)

- ✓ In $f_v(D)$ fitting, for ~65% of the drop spectra the KS test rejects all the selected models.
- ✓ This high rejection rate can be justified by the **large sample size (M)**.

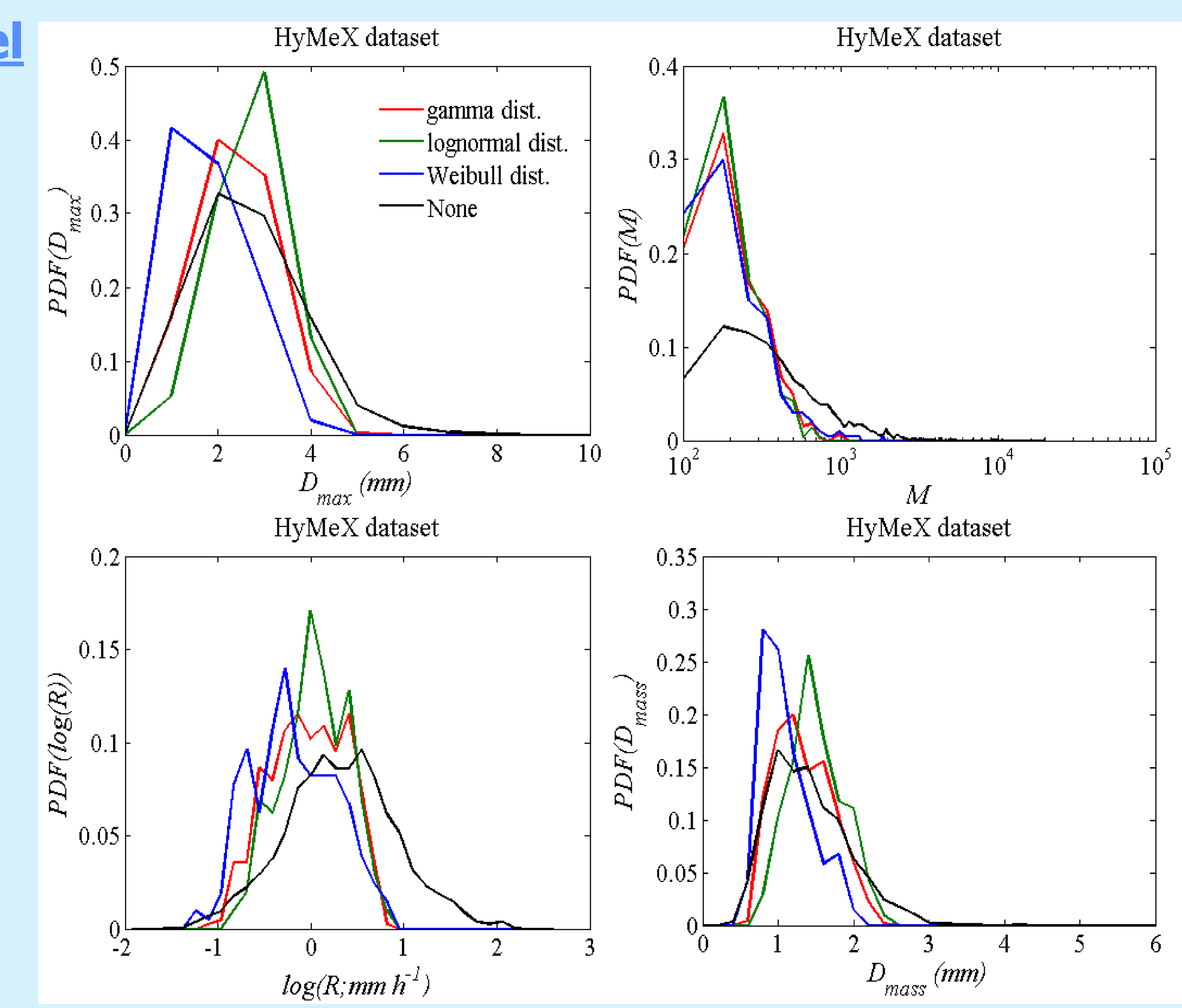
Fitting of $f_v(D)$	HyMeX	MC3E	IFloodS	IPHEx
M < 200	39.6%	42.0%	52.0%	39.5%
200 ≤ M < 500	61.4%	59.9%	69.1%	56.1%
500 ≤ M < 1000	89.8%	85.8%	91.0%	83.7%
1000 ≤ M < 2500	98.0%	98.6%	99.0%	98.2%
M > 2500	100%	100%	100%	100%

Example of measured 1-min. sample along with the three fitted distributions



Conditions leading a model to overcome the others

- ✓ D_{max}, R, and D_{mass} influences the selection of best model:
  - The lognormal distribution (heavy-tailed) represents better samples with high D_{max}, R, and D_{mass};
  - the opposite is valid for the Weibull distribution (a light-tailed distribution).
- ✓ The number of drops in 1 minute (M) does not affect the selection of the best model
  - For large M, none of the models is adequate for fitting
  - The same happens also for smaller M in a significant number of cases



More in:

Adirosi, E., Baldini, L., Lombardo, F., Russo, F., Napolitano, F., Volpi, E., Tokay, A. (2015). *Comparison of different fittings of drop spectra for rainfall retrievals. Advances in Water Resources*, *83*, 55-67.

Adirosi, E., Lombardo, F., Volpi, E , Baldini, L., (2016). *Raindrop size distribution: Fitting performance of common theoretical models, Advances in Water Resources*, *96*, 290-305,.